

of the post (*ku*) and the shadow length (*kou*) by their own values, add (*ping*) the squares, and take the square root (*khai fang chhu chih*) of the sum.^a This is

$$c = \sqrt{(a^2 + b^2)}.$$

Elsewhere expressions such as the square root of 5 are given to the nearest integer 'and a bit' (*yu chi*¹). There is an idea of arithmetical series because the declination circles are said to be each 19,833 *li* apart, and in each fortnightly period (the 24 *chhi*²) (see p. 405 below) the augmentation and diminution of the shadow is 9 *tshun* 9 $\frac{1}{8}$ *fên*.

The discussion of the right-angled triangle occurs at the beginning, in the oldest part of the text. It is worth reproducing in full (see Fig. 50), on account of its rude antiquity.^b

(1) Of old, Chou Kung addressed Shang Kao, saying, 'I have heard that the Grand Prefect (Shang Kao) is versed in the art of numbering. May I venture to enquire how Fu-Hsi anciently established the degrees of the celestial sphere? There are no steps by which one may ascend the heavens, and the earth is not measurable with a foot-rule. I should like to ask you what was the origin of these numbers?'

(2) Shang Kao replied, 'The art of numbering proceeds from the circle (*yuan*³) and the square (*fang*⁴). The circle is derived from the square^c and the square from the rectangle (lit. T-square or carpenter's square; *chü*⁵).

(3) The rectangle originates from (the fact that) $9 \times 9 = 81$ (i.e. the multiplication table or the properties of numbers as such).^d

(4) Thus, let us cut a rectangle (diagonally), and make the width (*kou*⁶) 3 (units) wide, and the length (*ku*⁷) 4 (units) long. The diagonal (*ching*⁸) between the (two) corners will then be 5 (units) long. Now after drawing a square on this diagonal, circumscribe it by half-rectangles like that which has been left outside, so as to form a (square) plate. Thus the (four) outer half-rectangles of width 3, length 4, and diagonal 5, together make (*tê chhêng*⁹) two rectangles (of area 24); then (when this is subtracted from the square plate of area 49)

弦圖

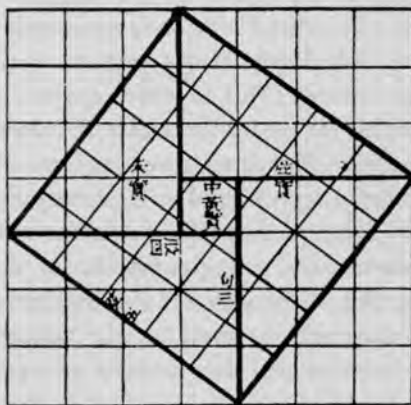


Fig. 50. The proof of the Pythagoras Theorem in the *Chou Pei Suan Ching*.

^a Ch. 1, p. 9a (Biot (4), p. 606).

^b The numbering of the paragraphs was introduced by Wylie (4). Cf. Mikami (16).

^c Presumably the writer was thinking of the diameter of a circle as equal to the diagonal of its inscribed square; perhaps also of exhaustion methods for getting π .

^d Chao Chün-Chhing explains that it is necessary to know the properties of numbers before one can work with geometrical figures. Note how radically different this is from the Euclidean method, in which actual numerical values are irrelevant provided the basic axioms and postulates are accepted. Here an arithmetical square is significantly given.

¹ 有奇

² 氣

³ 圓

⁴ 方

⁵ 矩

⁶ 句

⁷ 股

⁸ 徑

⁹ 得成

the remainder (*chang*¹) is of area 25. This (process) is called "piling up the rectangles" (*chi chü*²).^a

(5) The methods used by Yü the Great in governing the world were derived from these numbers.'

It will be remembered that the legendary Yü was the patron saint of hydraulic engineers and all those concerned with water-control, irrigation and conservancy. Epigraphic evidence from the Later Han, when the *Chou Pei* had taken its present form, shows us, in reliefs on the walls of the Wu Liang tomb-shrines (c. +140), the legendary culture-heroes Fu-Hsi and Nü-Kua holding squares and compasses (see Fig. 28 in Vol. 1, p. 164). The reference to Yü here undoubtedly indicates the ancient need for mensuration and applied mathematics.

(6) Chou Kung exclaimed, 'Great indeed is the art of numbering. I would like to ask about the Tao of the use of the right-angled triangle (lit. T-square).'^b

(7) Shang Kao replied, 'The plane right-angled triangle (laid on the ground) serves to lay out (works) straight and square (by the aid of) cords. The recumbent right-angled triangle serves to observe heights. The reversed right-angled triangle serves to fathom depths. The flat right-angled triangle is used for ascertaining distances.

(8) By the revolution of a right-angled triangle (compasses) a circle may be formed. By uniting right-angled triangles squares (and oblongs) are formed.

(9) The square pertains to earth, the circle belongs to heaven, heaven being round and the earth square.^c The numbers of the square being the standard, the (dimensions of the) circle are (deduced) from those of the square.^d

(10) Heaven is like a conical sun-hat (*li*³). Heaven's colours are blue and black, earth's colours are yellow and red. A circular plate is employed to represent heaven, formed according to the celestial numbers; above, like an outer garment, it is blue and black, beneath, like an inner one, it is red and yellow. Thus is represented the figure of heaven and earth.^e

(11) He who understands the earth is a wise man, and he who understands the heavens is a sage. Knowledge is derived from the straight line.^f The straight line is derived from the right angle.^g And the combination of the right angle with numbers is what guides and rules the ten thousand things.'

(12) Chou Kung exclaimed 'Excellent indeed!'^h

No further commentary is needed on this classical passage, except by way of emphasis on what seems a deeply significant point, namely, the statement in sentence (3) that geometry arises from mensuration. As has already been indicated, this seems to show the Chinese arithmetical-algebraic mind at work from the earliest times, apparently

^a Note the use of the same word, *chi*, found elsewhere in ancient writings meaning condensation or agglomeration (Vol. 2, pp. 41 etc.). In the translation of this vital passage on the Pythagoras theorem we have adopted the interpretation of Mr Arnold Koslow. We believe that the numbers given in the text were intended only as typical examples of the lengths of the three sides, each of which has its special technical term.

^b Biot (4), following commentators, translated this *chü*⁴ as gnomon; this emphasises the astronomical connection. The sense of the text makes it necessary to render the word in more than one way.

^c There was a good deal of speculation behind this, for 3, the number of heaven and the nearest approximation to π , was a male or Yang number, while 4, the number of earth, was female or Yin.

^d Cf. paragraph (2) above.

^e Although *li*³ is here in question (whatever it was), I cannot help suspecting a reference to the diviner's board (*shih*⁵), with its two plates, one round and one square; cf. Sect. 26*i* below.

^f The shadow.

^g Or gnomon.

^h Tr. auct.; adjuv. Wylie (4), Biot (4), Mikami (1).