

Mean Proportional

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What should the plural of my title be: “mean *proportionals*,” or “*means* proportional”? The singular “mean proportional” dates back to the sixteenth century; so does the use of each component word as a noun, and the plural term “middle proportionals.” I find “mean proportionals” in the eighteenth century, “proportional means” in the seventeenth, and “means geometrical” in the sixteenth, but never *means proportional*, even though it agrees with Greek usage, where the singular μέση ἀνάλογον becomes the plural μέσαι ἀνάλογόν.

Details are below, where I also remark on such peculiarities of grammatical number in English as are illustrated by George Eliot’s *Middlemarch* and by Ben Jonson’s confusion of subjunctive verbs with plural ones. I look at the Greek of Aristotle, as well as Euclid and Archimedes.

English number

There was a story in *The Onion* in 2000, “William Safire Orders Two Whoppers Junior” [27]. This is funny because:

- English sometimes imitates French by placing the adjective *after* the noun.
- English differs from French by *not* modifying adjectives to agree with nouns in number.
- French does not put a plural adjective with a singular noun.
- Therefore English should not do it.
- Only a pedant would insist on this rule.

Perhaps it is not pedantic to follow custom. Perhaps also these notes comprise pedantry galore.

That last word, “galore”: the 1995 *Concise Oxford Dictionary* calls it an adverb [30]; the 1981 *American Heritage Dictionary*, adjective [21]. Both dictionaries derive the word from an Irish prepositional phrase, while reporting that it follows what it modifies.

In seeming violation of the rule that adjectives as such are not made plural, we have the examples of *this* point and *that*, but *these* points and *those*. Perhaps the modifiers here should be called *determiners* (as in the

1995 *COD*), rather than adjectives (as in the *AHD*; the 1976 *COD* calls them *demonstrative* adjectives [29]).

One might say that “either” and “any” are determiners singular in number, while the plurals are “both” and “all”; but then “any” is plural in another sense of number, and “either” is *dual* in the same sense.

English can assign both a singular and a plural *verb* to the same subject, even in the same sentence:

The Garth family, which was rather a large one, for Mary had four brothers and one sister, were very fond of their old house, from which all the best furniture had long been sold.

Thus George Eliot in chapter 24 of *Middlemarch* [8, p. 200], with my underlining (as throughout these notes). Mrs Garth teaches her children out of the grammar of Lindley Murray, according to whom [23, p. 147],

A noun of multitude, or signifying many, may have a verb or pronoun agreeing with it, either of the singular or plural number; yet not without regard to the import of the word, as conveying unity or plurality of idea: as, “The meeting *was* large;” “The parliament *is* dissolved;” “The nation *is* powerful;” “My people *do* not consider: *they* have not known me;” “The multitude eagerly *pursue* pleasure, as *their* chief good;” “The council *were* divided in *their* sentiments.”

Having alluded to this passage in what *I* quoted, Eliot has her own character quote Murray [8, p. 202]:

“Now let us go through that once more,” said Mrs. Garth, pinching an apple-puff which seemed to distract Ben, an energetic young male with a heavy brow, from due attention to the lesson. “‘Not without regard to the import of the word as conveying unity or plurality of idea’—tell me again what that means, Ben.”

(Mrs. Garth, like more celebrated educators, had her favorite ancient paths, and in a general wreck of society would have tried to hold her “Lindley Murray” above the waves.)

“Oh—it means—you must think what you mean,” said Ben, rather peevishly. “I hate grammar. What’s the use of it?”

Grammar has a bad use, if it is to blame for a 2016 headline, “55 Years After Their Honeymoon Road Trip, This Couple is Taking Their Old VW Bug on One Last Adventure” [19]. The writer must have thought there was a rule whereby “couple,” being singular, needed a singular verb, even if not a singular pronoun. To my mind, the natural thing would be to say that the couple *are* taking their bug on an adventure. Would the headline writer say that a couple of people *is* talking? If that’s different, because “people” is plural, well, it is *formally* singular, just as “couple” is.

As George Eliot mixes singular and plural, so too indicative and subjunctive [8, p. 219]:

She would follow him out of the room and put her hand on his arm moaning out, “Save my boy.” Once she

pleaded, "He has always been good to me, Mr. Lydgate: he never had a hard word for his mother,"—as if poor Fred's suffering were an accusation against him, all the deepest fibres of the mother's memory were stirred, and the young man whose voice took a gentler tone when he spoke to her, was one with the babe whom she had loved, with a love new to her, before he was born.

If there is an error here though, it is probably the publisher's; in place of "against him, all . . ." the Project Gutenberg edition of *Middlemarch* has "against him. All . . ."

In "Fred's suffering were an accusation" though, Ben Jonson would say the verb was not subjunctive, but *plural* [17, pp. 65–6]:

Nounes signifying a multitude, though they be of the singular number, require a verbe plurall.

Lidgate, lib. 2:

*And wise men rehearsen in sentence
Where folke be drunken, there is no resis-
tance.*

This exception is in other nounes also very common; especially when the verbe is joyned to an adverbe, or conjunction: *It is preposterous to execute a man, before he have beene condemned.*

Gower, lib. 1:

*Although a man be wise himselve,
Yet is the wisdome more of twelue.*

Chaucer:

*Therefore I read you this counsell take,
Forsake sinne, ere sinne you forsake.*

In this exception of number, the verbe sometime agreeth not with the governing nounce of the plurall number, as it should, but with the nounce governed: as, *Riches is a thing oft-times more hurtfull, then profitable to the owners.* After which manner the Latines also speake: *omnia pontus erat.*

I learned of Jonson's analysis from the 1882 doctoral study of the English subjunctive by Gerold Hotz [14, pp. 1–2], who reviews how

the English grammarians of past centuries either ignored its existence as a mood, or misconceived it . . .

BEN JONSON, after having stated that collective nouns require a verb in the plural (as f. i. *folke*), goes on to say, that “this exception (i. e. plural for sing.) is in other names very common, especially when the verb is joined to an adverb or conjunction” and sees such cases in *although a man be wise; ere sinne you forsake; before he have been condemned!!* where the subj. is explained as being the p l u r a l ! of those respective verbs.

I have tried to preserve Hotz's peculiar typography (including the missing period in what was presumably mean to be read “as for instance *folke*”). It is an interesting question whether people such as Jonson can actually be mistaken in their analysis of their own speech.

I learned of Hotz from *An Old English Grammar and Exercise Book* [28, p. 71], where Alphonso Smith quotes him on the use of the subjunctive for indirect speech [14, p. 89]:

As to whether the statement refer to a fact or not, whether the subject-matter be vouched by the reporter, as regards its objective reality and truth, the subj. does not tell. It simply represents a statement as reported.

Meanwhile, as far as our original question is concerned, it would seem to be legal custom to speak of *attorneys general* and *courts martial*. Fowler gives the latter example [13], along with *Lords Justices*. Here perhaps both words are construed as nouns. According to Wikipedia, there were deputies called Lords Justices General and General Governors of Ireland.

Greek number

If there are justices general, there can be means proportional. Archimedes refers to them in *De Sphaera et Cylindro II* [1, p. 192], when he analyzes the problem of finding a sphere that is equal to a given cone or cylinder:

δύο ἄρα δοθεισῶν εὐθειῶν τῶν ΓΔ, ΕΖ δύο μέσαι ἀνάλογόν
εἰσιν αἱ ΗΘ, ΜΝ. δοθείσα ἄρα ἐκάτερα τῶν ΗΘ, ΜΝ.

Here, as I understand,

- μέσαι is the feminine *plural* nominative of the adjective μέσος, -η, -ον, complementing αἱ ΗΘ, ΜΝ, which is feminine because the omitted γραμμαί “lines” is feminine (I wrote elsewhere about the convenience of gendered articles [25], citing Netz [24]);
- ἀνάλογόν (so accented because εἶσω follows) is a form of the adjective ἀνάλογος, -ον; it could be a neuter nominative, or an accusative of any gender, but it is *singular*, and this is a sign to construe it adverbally.

The sense of the passage is that $H\Theta$ and MN are *means* in the *proportional* sense—*proportionally*—of $\Gamma\Delta$ and EZ ; symbolically,

$$\Gamma\Delta : H\Theta :: H\Theta : MN :: MN : EZ.$$

We can also say that $H\Theta$ and MN are *geometric* means of $\Gamma\Delta$ and EZ , rather than, say, *arithmetic* means.

Nonetheless, Netz translates the passage [2, p. 189],

therefore $H\Theta$, MN are two mean proportionals between two given lines, $\Gamma\Delta$, EZ ; therefore each of (the lines) $H\Theta$, MN are given.

I think Netz’s last “are” is a mistake, but each of the lines *is* given, since “each” is singular, like *ἐκάτερα*, and it does not matter that “lines” is plural (just as, if “a

couple is” were correct, then so would “a couple of people is” be—but it’s not). In any case, “two mean proportionals” *may* reflect the usage of Heath and other writers in English on geometry.

The enunciation and conclusion of Proposition 13 of Book VI of the *Elements* of Euclid ([10, p. 110] or [12]) are,

Δύο δοθεισῶν εὐθειῶν μέσην ἀνάλογον προσευρεῖν.

Δύο ἄρα δοθεισῶν εὐθειῶν τῶν AB, ΒΓ μέση ἀνάλογον προσεύρηται ἡ ΔΒ· ὅπερ ἔδει ποιῆσαι.

Here *μέσην* and *μέση* (“mean”) are respectively accusative and nominative, but *ἀνάλογον* (“proportional”) does not change. Heath’s translation [11, II p. 216]:

To two given straight lines to find a mean proportional.

Therefore to the two given straight lines *AB, BC* a mean proportional *DB* has been found.

Under “*ἀνάλογος, ον,*” the LSJ lexicon [18] discusses the adverbial use of *ἀνάλογον* in Aristotle, where the word has the meaning of the prepositional phrase *ἀνά λόγον*. The first example is from *Nicomachean Ethics* VIII.vi.6, 1158^a33–6 [4]; since it is of independent interest, I give it here, glossed with my proposed translation:

ἡδὺς δὲ καὶ χρήσιμος pleasant and useful
 ἅμα at the same time,
 εἴρηται ὅτι ὁ σπουδαῖος · it is said that the good man [is];
 ἀλλ' ὑπερέχοντι but to the one who exceeds [him]
 οὐ γίνεται does not become—
 ὁ τοιοῦτος φίλος, such a man—a friend,
 εἰ μὴ καὶ if not also
 τῇ ἀρετῇ ὑπερέχεται · exceeded in virtue;
 εἰ δὲ μή, if not,
 οὐκ ἰσάζει he does not equalize
 ἀνάλογον proportionally
 ὑπερεχόμενος. being exceeded.

My interpretation accords with Rackham's [3, p. 475]:

The good man, as we have said, is both useful and pleasant, but the good man does not become the friend of a superior, unless his superior in rank be also his superior in virtue; otherwise the good man as the inferior party cannot make matters proportionally equal.

Such a translation is what “more recent commentators suggest,” according to Bartlett and Collins, who themselves “follow Aspasius’s generally accepted interpretation of this passage,” which is opposite [5, p. 173]:

It has been said that the serious person is at once pleasant and useful; yet such a person does not become a friend to someone who exceeds him [in power], unless [the person in power] is also exceeded [by the serious person] in virtue. But if this does not occur, [the serious person] is not rendered equal [to the person of

greater power], since he is exceeded in the relevant proportion.

It is not clear how to resolve the question of what Aristotle means here.

Mathematics

It is clear how to resolve a *mathematical* question. At least it is clear how to stake out a claim: use deductive logic. There can be no *general* method for identifying which claims to make in the first place, though there is a method in certain restricted domains [26].

At present we are dealing with a *grammatical* or *etymological* question that happens to arise in mathematics.

Elements VIII, Proposition 12, is enunciated:

Δύο κύβων ἀριθμῶν δύο μέσοι ἀνάλογόν εἰσιν ἀριθμοί, καὶ ὁ κύβος πρὸς τὸν κύβον τριπλασίονα λόγον ἔχει ἥπερ ἡ πλευρὰ πρὸς τὴν πλευράν.

As I would put it,

Between two cube numbers are two proportionally mean numbers, and the cube has to the cube triple the ratio that the side [has] to the side.

Still, for Heath, there are “two mean proportional numbers.”

I have not been able to find “mean proportional” as a *defined* term in the *Oxford English Dictionary* [22]. In the articles on the component words, I find the phrase itself only under “Proportional,” in a quotation from Hutton’s *Course of Mathematics*, dated 1798:

The mean proportional of two numbers is the square root of their product.

Hutton’s book seems to have gone through many editions; in an 1812 version [15, p. 117], I find:

PROBLEM I.

To find One Geometrical Mean Proportional between any Two Numbers.

MULTIPLY the two numbers together, and extract the square root of their product, which will give the mean proportional sought . . .

PROBLEM II.

To find Two Geometrical Mean Proportionals between any Two Numbers.

DIVIDE the greater number by the less, and extract the cube root . . .

The same edition reproduces another *OED* quotation, making clear that “proportional” is indeed used as a noun [15, p. 110]:

So, the four proportionals, 4, 2, 6, 3 are set thus, 4 : 2 :: 6 : 3, which means, that 4 is to 2 as 6 is to 3; or thus, $4 : 2 = 6 : 3$, or thus, $\frac{4}{2} = \frac{6}{3}$, both which mean, that the ratio of 4 to 2, is equal to the ratio of 6 to 3.

The earliest quoted *mathematical* use of “proportional” as an *adjective* is from Billingsley’s 1570 translation of Euclid’s *Elements* [9, V, fol. 130 v.]:

Magnitudes which are in one and the selfsame proportion, are called Proportionall.

Here and elsewhere, in place of Billingsley’s “proportion,” Heath has “ratio”; the Greek of the quoted definition is,

Τὰ δὲ τὸν αὐτὸν ἔχοντα λόγον μεγέθη ἀνάλογον καλεῖσθω.

There is a plural subject, τὰ μεγέθη, but what these are called, ἀνάλογον, is singular, unless indeed the word be construed as an adverb.

The *OED*’s earliest quoted use of “proportional” as a noun is from John Dee’s “Mathematicall Praeface” to Billingsley’s work [6, c.iiij v.]:

Betwene two lines giuen, finde two middle proportionals, in Continuall proportion: by the hollow Parallelipipedon, and the hollow Pyramis, or Cone.

Here “middle” could presumably be “mean.”

The *OED* defines a *mathematical mean* as the term, or one of the terms, between the first and last of a progression. The mean can be arithmetic, geometric, or harmonic. The first quotation is from the 1571 *Pantometria* of Leonard Digges [7, p. 150]; it is from the fourth definition below, although we want to see the third too:

Diffinitions. PProportion is a mutuall or enterchangeable relation of two magnitudes, being of one kind, compared together in respecte of their quantities.

The second dffinition. When the proportion of two magnitudes is such as may be expressed with numbers, then is it certaine & apparant and here is called rational: But when the proportion is such as cannot be expressed with numbers, but with their rootes onely, then is that proportion certayne also, but not apparante, and therefore here I name it surde or irrationall.

The thirde diffinition. When there be thrée suche magnitudes or quantities that the first to the second retayne the same proportion that the second doth to the third, those quantities are saide to be proportionall, and the first to the thirde retayneth double the proportion of the first to the second, and the seconde is named meane proportionall betwéene the first and the last.

The fourth Diffinition When foure magnitudes are likewise in continual proportion, the first & the

fourth are the extremes, and the second and thirde the meanes, and the extreames are sayd to haue triple the proportion of the meanes.

The fifth diffinition. Any lyne or number is sayde to be diuided by extreame & meane proportion, when the diuision or section is suche or so placed, that the whole line or number retayne the same proportion to the greater parte, that the greater doth to the lesser.

The usage here would seem to sanction both *mean proportionals* and *means proportional*. Nonetheless, according to a search of the electronic transcription, Digges never refers to more than one mean proportional as such. He refers to *means geometrical*, although these seem to be *methods* rather than magnitudes [7, p. 189]:

for the farther satisfaction of such as seeke to reach beyond the commoⁿ sort, and vvill not content them selues vvith bare rules and preceptes, vnlesse they may also conceiue some grounde and reason of their vvorkings, I haue thought good to euery of these Problemes ensuing, to adioyne his peculiare figure, vvith meanes Geometricall (no regarde had to Irrationall numbers vvithout aide of Arithmetically supputation) to searche out the sides, Diameters, and Axis, of al the regular bodies inscribed or circumscribed of spheres, by knowvledge of their Diameters, or mutually coⁿ-ferred together by knowvledge of some side.

In any case, perhaps Digges's work is what Merriam-Webster alludes to, after defining "mean proportional"

as geometric mean; the dictionary says [20],

The first known use of *mean proportional* was in 1571.

Meanwhile, under the mathematical definition of “mean,” the *OED* quotes also from an example in the 1696 ΛΟΓΙΣΤΙΚΗΛΟΓΙΑ of Jeake, and here the word is both plural and modified by “proportional” [16, p. 570]:

As if between 2 and 54, two proportional Means be sought, the Lesser will be 6 and the Greater 18: For 54 multiplied by 4, the Square of 2, produceth 216, whose Cube Root is 6 the lesser Mean: and 2916 the Square of 54 multiplied by 2 produceth 5832, whose Cube Root is 18 the greater Mean; so if 2 divide 54 the Cube Root of 27, the Quotient is 3, which multiplying 2 is 6, and 6 is 18.

However, in his next example, Jeake uses also “mean proportional”:

To get 3 Means between 2 Numbers given, proceed by the first Case to get the middle Proportional between the 2 given Extreams; then between that middle Proportional and the least Extream, a mean Proportional is to be found in like manner, and also between the greatest Extream and that middle Proportional.

English resists consistency.

References

- [1] Archimedes. *Archimedis Opera Omnia Cum Commentariis Eutochii*, volume I. B. G. Teubner, 1880. Recension of the Florentine Codex, with Latin translation and notes by J. L. Heiberg.
- [2] Archimedes. *The Two Books On the Sphere and the Cylinder*, volume I of *The Works of Archimedes*. Cambridge University Press, Cambridge, 2004. Translated into English, together with Eutocius' commentaries, with commentary, and critical edition of the diagrams, by Reviel Netz.
- [3] Aristotle. *Nicomachean Ethics*, volume 73 of *Loeb Classical Library*. Harvard University, revised edition, 1934. With an English translation by H. Rackham. First edition 1926.
- [4] Aristotle. *Nikomakhos'a Etik*, volume 01 of *Klasik Metinler [Classic Texts]*. Ayraç, Ankara, 1997. Facsimile of the 1890 Greek text of I. Bywater with Turkish translation by Saffet Babür.
- [5] Aristotle. *Nicomachean Ethics*. University of Chicago, 2011. Translated, with an interpretive essay, notes, and glossary, by Robert C. Bartlett and Susan D. Collins.
- [6] John Dee. Mathematicall preface. In *The Elements of Geometrie of the most auncient Philosopher Euclid of Megara* [9]. www.gutenberg.org/files/22062/22062-h/22062-h.htm, accessed March 3, 2025.
- [7] Leonard Digges. *A geometrical practise, named Pantometria diuided into three bookes, longimetra, planimetra, and stereometria*. Henrie Bynneman, London, 1571.

quod.lib.umich.edu/e/eebo/A20458.0001.001, accessed March 3, 2025; archive.org/details/bim_early-english-books-1475-1640_a-geometrical-practise-digges-leonard_1571, accessed March 11, 2025.

- [8] George Eliot. *Middlemarch*. Wordsworth Classics, 2000. First published serially, 1871–2.
- [9] Euclid. *The Elements of Geometrie of the most auncient Philosopher Euclid of Megara. Faithfully (now first) translated in the Englishe toung, by H. Billingsley, Citizen of London*. Iohn Daye, London, 1570. archive.org/details/elementsgeometr00eucl, accessed March 3, 2025.
- [10] Euclid. *Euclidis Elementa*, volume II of *Euclidis Opera Omnia*. Teubner, Leipzig, 1884. Edited with Latin interpretation by I. L. Heiberg. Books V–IX.
- [11] Euclid. *The Thirteen Books of Euclid’s Elements*. Dover Publications, New York, 1956. Translated from the text of Heiberg with introduction and commentary by Thomas L. Heath. In three volumes. Republication of the second edition of 1925. First edition 1908.
- [12] Euclid. Euclid’s Elements – home page – original Greek text. users.ntua.gr/dimour/euclid/, 2020. Edited by Dimitrios E. Mourmouras. Accessed June 18, 2024.
- [13] H. W. Fowler. *A Dictionary of Modern English Usage*. Oxford University Press, London, 1926. Corrected reprint of 1954.
- [14] Gerold Hotz. *On the Use of the Subjunctive Mood in Anglo-Saxon, and its Further History in Old English*. S. Höhr,

Zürich, 1882. “An inaugural dissertation written for the degree of doctor of philosophy and presented to the philosophical faculty of the University of Zürich”.

- [15] Charles Hutton. *A Course of Mathematics*, volume I. Samuel Campbell et al., New York, 1812. In two volumes. For the use of academies, as well as private tuition. From the fifth and sixth London editions. Revised and corrected by Robert Adrian. archive.org/details/acoursemathemat08huttgoog, accessed March 2, 2025.
- [16] Samuel Jeake. *ΛΟΓΙΣΤΙΚΗΛΟΓΙΑ, or Arithmetick Surveighed and Reviewed*. Walter Kettilby, London, 1696. archive.org/details/bim_early-english-books-1641-1700_-or-ari_jeake-samuel_1696, accessed March 3, 2025.
- [17] Ben Jonson. *The English Grammar Made by Ben Jonson: For the benefit of all Strangers, out of his observation of the English Language now spoken, and in use*. Lanston Monotype Corporation, London, 1928. With prefatory note by Strickland Gibson. Reprint of the 1640 edition. archive.org/details/englishgrammarm0000jons, accessed March 6, 2025.
- [18] Henry George Liddell and Robert Scott. *A Greek-English Lexicon*. Clarendon Press, Oxford, 1996. “Revised and augmented throughout by Sir Henry Stuart Jones, with the assistance of Roderick McKenzie and with the cooperation of many scholars. With a revised supplement.” First edition 1843; ninth edition 1940.
- [19] Jordi Lippe and Jordi Lippe-McGraw. 55 years after their honeymoon road trip, this couple is taking their old

- VW bug on one last adventure. *Travel + Leisure*, March 10 2016. web.archive.org/web/20190930233319/http://www.travelandleisure.com/articles/couple-takes-final-road-trip-in-beetle, accessed March 3, 2025.
- [20] Merriam-Webster. Mean proportional. In *Merriam-Webster.com dictionary*. Merriam-Webster, n.d. www.merriam-webster.com/dictionary/mean%20proportional, accessed March 3, 2025.
- [21] William Morris, editor. *The Grolier International Dictionary*. Grolier, Danbury, Connecticut, 1981. Two volumes. Appears to be the *American Heritage Dictionary* in a different cover.
- [22] James A. H. Murray et al., editors. *The Compact Edition of the Oxford English Dictionary*. Oxford University Press, 1971. Complete text reproduced micrographically. Two volumes. Original publication, 1884–1928.
- [23] Lindley Murray. *English Grammar*. Collins and Perkins, New York, 1809. archive.org/details/englishgrammara12murrgoog, accessed March 3, 2025.
- [24] Reviel Netz. *The Shaping of Deduction in Greek Mathematics*, volume 51 of *Ideas in Context*. Cambridge University Press, Cambridge, 1999. A study in cognitive history.
- [25] David Pierce. Abscissas and ordinates. *J. Humanist. Math.*, 5(1):223–264, 2015. dx.doi.org/10.5642/jhummath.201501.14.
- [26] David Pierce. On gödel’s incompleteness theorem. *Journal of Humanistic Mathematics*, July 2025. To appear; meanwhile

at polytropy.com/wp-content/uploads/2025/03/goedel-incompleteness.pdf.

- [27] Robert Siegel, editor. *The Onion: Dispatches From the Tenth Circle. The Best of the Onion*, chapter William Safire Orders Two Whoppers Junior, page 8. Three Rivers Press, New York, 2001. Satire dated September 20, 2000. theonion.com/william-safire-orders-two-whoppers-junior-1819565735/, accessed March 3, 2025.
- [28] C. Alphonso Smith. *An Old English Grammar and Exercise Book*. Allyn and Bacon, Boston and Chicago, 1898. With inflections, syntax, selections for reading, and glossary. New edition, revised and enlarged. First edition 1896.
- [29] J. B. Sykes, editor. *The Concise Oxford Dictionary of Current English*. Oxford, sixth edition, 1976. Tenth impression 1980. First edited by H. W. Fowler and F. W. Fowler.
- [30] Della Thompson, editor. *The Concise Oxford Dictionary of Current English*. Clarendon Press, Oxford, ninth edition, 1995. First edited by H. W. Fowler and F. W. Fowler.